An interesting fact brought out in the observations on this slope is that the highest average minimum temperature was found at the 225-foot station, 50 feet below the summit of the ridge; the minimum temperature at the summit was as high as that at the 225-foot station on only one night during the season. The summit of the ridge is well rounded and the ground sloped away from the summit station in all directions. The steep slope began about 20 feet from the thermometer shelter.

THE PRINCIPLE OF THE CONSERVATION OF ANGULAR MOMENTUM AS APPLIED TO ATMOSPHERIC MOTIONS.

H. W. CLOUGH.

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The change in the relative easterly velocity of a body moving freely without friction on the earth's surface, due to its variation in latitude, sometimes referred to as the change of velocity with change of latitude, was first imperfectly recognized by Hadley about 1735, who explained the trade winds in this manner. He assumed that a body tends to retain its initial absolute gyratory velocity when forced to another parallel. Ferrel, however, showed that the law of conservation of angular momentum or the preservation of areas applies to a body on the earth's surface, which even at relative rest is gyrating around the earth's axis with a velocity proportional to the cosine of the latitude. Such a body, on being forced to another latitude, tends to preserve its angular momentum, hence must change its absolute east velocity inversely as the cosine of the latitude. A body then moving toward the pole would acquire a relative easterly velocity. The formula for the increase in relative easterly velocity in a body moving from lat. ϕ to lat. ϕ_1 is $V_1 = 465$ m. p. s. $\left(\dfrac{\cos^2\phi}{\cos\phi_1} - \cos\phi_1 \right)$. For example, a body at rest in lat. 70° would acquire at lat. 80° a relative easterly velocity of 232.6 m. p. s.

It is well known, however, that the enormous velocities thus called for by the principle of equal areas do not exist in the atmosphere, and since this principle is of rigorous application to all air motions, the problem is to account for the moderate velocities actually observed.

Hadley accounted for the difference between the theoretical and the actual strength of the trade winds by the retarding influence of friction with the earth's surface.

Ferrel[1] arrives at the following theoretical west-east velocities for a frictionless atmosphere on a smooth rotating globe with a decrease in temperature from the equator to the poles. The plus signs are the easterly velocities; the minus signs, westerly velocities.

Latitude	Velocity per hour.	Latitude.	Velocity per hour.
	Miles.		Miles.
90°	∞	35° 16'	0
-80°	+3807	30°	-100
70°	+1669	20°	-239
60°	+865	10°	-320
50°	+410	0°	-346
40°	+108		

He attempted to show that these excessive velocities, required by the so-called "canal theory," could be re-

duced to moderate values by introducing proper coefficients of friction. He states ("A Popular Treatise on the Winds," p. 118, par. 81):

The velocities of the east and west components at the surface, depending upon the forces of momentum, depend very much upon the nature of that surface. For the velocities increase until the resistances to the motions are equal to the forces producing them, so that the smoother the surface and the less the friction between the air and that surface the greater the velocities. If there were no resistance from the earth's surface these velocities would be very great, but on account of the smallness of the forces giving rise to them and maintaining them by overcoming friction when they are once established, these velocities are comparatively very small.

Oberbeck's theory likewise calls for excessive velocities in middle latitudes.

Friction, furthermore, is to be considered since without it the atmospheric currents under the continuous influence of accelerating forces would attain to indefinitely great velocities. (Abbe's translation, The Mechanics of the Earth's Atmosphere, *Smithsonian Misc. Col.*, Vol. 34, Art. X.

Von Helmholtz' theory of atmospheric circulation introduced resistances due to vortex motions, which he conceived as originating from the mutual interaction of air currents. One of his assumptions is that the internal friction of the atmosphere, in the higher levels at least, is practically negligible.

Bigelow's theory involves horizontal counter currents in the lower levels from the surface to 2 or 3 kilometers, facilitating interchange of polar and equatorial temperatures and tending by the formation of vortices to retard the upper circulation to the moderate velocities actually observed.

The validity of Ferrel's "canal theory" has been questioned by many since it involves a retardation of the upper currents far greater than could conceivably be accounted for by viscosity, surface friction, turbulence, vertical convection or vortex motions.

On the other hand the rapid upper east wind over the equator, or the "Krakatoa wind," by some[2] described as a wind without gradient, has been explained as a result of the transference of air from a position of relative rest at about latitude 14° or 15°, to the equator, the relative westward velocity being increased in accordance with the law of preservation of areas. Evidently in this case no retarding or damping influences are desirable.

Many meteorological textbooks make vague and misleading references to this subject. Hann (Lehrbuch der Meteorologie, 3d ed., p. 433) says:

According to the principle of the preservation of the moment of rotation, air passing from the equator toward higher latitudes acquires in this way extraordinarily great easterly components. Actually, friction, the mixing of air masses, etc., prevent the attainment of such velocities.

Angot (Traité de Élémentaire Météorologie, p. 154):

It may be shown that if friction were absent the air, acquiring a great relative easterly velocity, could not, under the effect of the centrifugal force which results from the relative velocity, remain in the vicinity of the pole.

Milham (Meteorology, p. 62):

The air masses will be deviated to the right in the northern hemisphere and become more and more a west current encircling the pole in a great whirl. As this ring of whirling air about the pole approaches it the mass remains constant, the radius is decreasing and thus the velocity must steadily increase and it has been computed that if the velocities were not held down by friction they would amount to hundreds of thousands of miles per hour.

[1] Values computed by Bigelow from Ferrel's equation. International Cloud Report, p. 603.

[2] Teisserenc de Bort, L. Report on the present state of our knowledge respecting the general circulation of the atmosphere, 1893, p. 5.

Moulton (An Introduction to Astronomy, p. 86):

If it were not for friction with the earth's surface, a mass of air moving from latitude 40° to latitude 45°, a distance of less than 350 miles, would acquire an eastward velocity with respect to the surface of the earth of over 40 miles per hour.

The impression gained by reading these extracts is that air moving from one latitude to another undergoes thereby a change in its velocity with reference to the earth's surface, while its velocity remains unchanged if it moves along a parallel. The increase in the velocity thus acquired is supposed to be reduced by friction so that only moderate velocities actually result.

Davis (Elementary Meteorology, p. 103) correctly pointed out the effect of a change in latitude on air masses. He says, referring to the trade winds:

The explanation still generally current follows that given by Hadley, in brief, as follows: If a mass of air moves from latitude 30° north toward the Equator, it advances upon latitudes whose eastward rotary velocity is greater and greater, and in consequence of this the air lags behind and, hence, appears as an oblique northeast surface wind. Indeed, if it were not for the friction with the surface of the land and sea, by which the advancing air continually acquires something of the eastward motion of the latitude that it enters, there should be a violent westward hurricane of a hundred miles or more at the Equator according to this theory.

This explanation contains two serious errors, which are here referred to because they have gained general currency. Hadley's explanation implies that there is no effect produced on motion to the east or west, while, as stated above, the deflective force arising from the earth's rotation is independent of the direction of motion. Again, Hadley's explanation teaches that a body moving toward the Equator continually lags westward, so that if friction had no effect it would attain a great velocity to the west when it reached the Equator. This is wrong. The lagging, if such an expression is introduced at all, can not be continually in one direction, as to the west, but can only be at *right angles to the momentary direction of motion*, and hence can produce *no effect on the velocity*. If a body were given a velocity of 25 miles an hour to the south when in latitude 30° north, and was supposed to move without friction over a level surface, it would continue to move at the same moderate rate whatever latitude it reached; while Hadley's explanation would give it a velocity of a hundred miles or more westward at the Equator. A proper understanding of the true value and action of the deflective force should therefore be introduced into the popular teaching of meteordlogy.

Notwithstanding the fact that Davis wrote this in 1893, many later writers have continued to give the same erroneous explanation of the movement of air masses in obeying the law of preservation of areas. This may be partly accounted for by a failure in Davis's criticism to discriminate clearly between *relative* velocity and easterly velocity.

In view of the divergence among theoretical meteorologists regarding this problem, it is not surprising that popular writers and nonmathematical students experience difficulty in correctly understanding the principles involved.

The problem is simplified by a preliminary consideration of the motion of a body moving without friction on the globe, under the deflecting force of the earth's rotation. The motion of an air particle is but little more complicated. The theory is simple but leads to rather complex mathematical formulæ for expressing the movement under varying conditions of velocity and latitude.

F. J. W. Whipple has given the most complete discussion of the paths of bodies moving without friction on a rotating globe. (Phil. Mag., vol. 33, p. 457.)

Take the simple case that the body moves in such a small range of latitude that the sine of the latitude which enters into the formula expressing the deflecting force may be regarded as constant. This formula is, for a unit mass of 1 gram, $2v\omega \sin \phi$, where v is the velocity of the body in centimeters per second, ω the angular velocity of the earth's rotation per second, and ϕ the latitude. Since this force always acts at right angles

to the momentary direction of motion of the body, the latter preserves its initial relative velocity, or its velocity with reference to the earth's surface, under all circumstances. If the sine of the latitude is constant, or nearly so, the deflective force is constant and the body being continually deflected from its course by equal angles in equal times describes a circle. Actually the circle is not quite closed and the path forms a series of overlapping loops along which the body gradually moves westward along a parallel. The path being circular, there must be equality between the deflecting force and the centrifugal force due to the inertia of the body. The latter force is proportional to $\frac{v^2}{r}$, v being the initial velocity and r the radius of curvature of its circular path. The deflecting force is $2\omega v \sin \phi$. Hence $r = \frac{v}{2\omega \sin \phi}$ or $\frac{6857v}{\sin \phi}$. The formula gives the radius of curvature in meters, if the velocity is expressed in meters per second. The table in Davis's Meteorology (p. 104) gives incorrect values of the radius of curvature corresponding to varying latitudes and velocities. The values should be divided by two to obtain correct value of r.
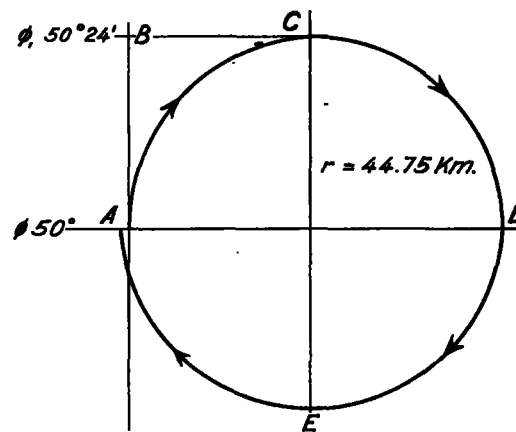

FIG. 1.

Figure 1 illustrates the approximately circular path of a body projected due north from a point A at latitude ϕ. It describes a circular path with a radius r reaching its extreme northern limit at C in latitude ϕ_1, where its motion is due easterly. Continuing to be deflected, it crosses latitude ϕ at D, moving due south and returns nearly to its starting point, crossing latitude ϕ slightly west of A.

Assume a body projected northward from a point on the fiftieth parallel with a velocity of 5 meters per second. Substituting in the formula above, r equals 44.75 kilometers. The latitude ϕ_1 reached at the most northerly point of its path where its motion is due easterly is 50° $\frac{4475}{11118}$, or 50° 24′ N., 111.18 kilometers being the length of a degree of latitude along the meridian at latitude 50°. The time of revolution is 15.7 hours. The cosine of 50° is 0.6428 and of 50° 24′ is 0.6374. Substituting in the above formula for change in easterly velocity, $v_1 = 465\left(\frac{0.6428^2}{0.6374} - 0.6374\right) = 5$ meters per second.

In this formula v_1 represents the change in easterly velocity in accordance with the law of areas, experienced by a body

moving from latitude ϕ to latitude ϕ_1. Easterly velocity signifies the easterly component of the relative velocity of the body. The relative velocity is the initial velocity of projection, irrespective of direction. If the body is projected due north, its easterly velocity is zero at the point of projection which is A in the figure. As it curves to the eastward the easterly component of the relative velocity gradually increases until when it reaches C its easterly velocity equals its relative velocity. In the above illustration, the body moving from latitude 50° to 50° 24′ acquires an easterly velocity in accordance with the law of preservation of areas of 5 meters per second in excess of that which it possessed at latitude 50°. The easterly velocity at A was zero. Hence at latitude 50° 24′ it has acquired an increased easterly velocity of 5 meters per second but its relative velocity is unchanged. Continuing to move southward, it loses its easterly velocity at D and gains a westerly velocity of 5 meters per second at E, the most southerly point reached.

Therefore a body moving freely under the influence of the earth's deflective force undergoes variations in its easterly velocity in strict accordance with the principle of the preservation of areas. Its relative velocity or initial velocity of projection remains unchanged. The deflective force acts perpendicularly to the momentary direction of motion, causing the body to describe a nearly circular path with unchanged initial relative velocity. A body moving freely over the earth's surface from one latitude to another can and does obey the law of equal areas without any change in its relative velocity. At the extreme latitude which it is able to reach in its curved path it gains or loses by the law of areas an easterly velocity precisely equivalent to its initial velocity, if the direction of projection is along a meridian.

In the above quotation from Davis the term velocity is used in one sense only, meaning relative velocity. Referring to Hadley's teaching that a body moving toward the Equator continually lags westward and would attain a great velocity to the west when it reached the Equator, he makes the criticism that this lagging can produce no effect on the velocity, meaning relative velocity. Hadley was correct in the sense that a change of latitude involves a change in the easterly velocity of a freely moving body, but he was wrong in assuming that a change in relative velocity would result. In

Davis's illustration a body given a velocity of 25 miles per hour to the southward at latitude 30° would not even reach latitude 28°, a fact which he omitted to state. In order that it may reach the equator it should have an initial southward velocity of 116 meters per second, and at the Equator it would be moving due westward with its initial velocity.

Thus moving air does not acquire the excessive velocities implied in the extracts given above, because it does not move over the required range of latitude. The increase in easterly velocity is checked when the limiting latitude is reached, hence friction is obviously not needed to reduce the excessive velocities.

In order that the body may continue in its original direction of projection, a force equal and opposite to the deflective force must act on the body. Assume that the body moves on a plane with an inclination to the left of the direction of projection. With the direction due north, the plane will slope to the west. If the inclination of the plane is such that the small component of gravity tending to draw the body down the plane is equal to the deflective force, the inclination of the plane gradually increasing with increasing latitude, the body will continue moving in a due north direction with uniform velocity.

The uniform motion of a body on an inclined plane is strictly analogous to that of air particles along straight isobars, neglecting friction. Consider isobars extending north and south with low pressure to westward and high pressure to eastward. The air particles will have a movement from south to north with uniform velocity under the equal and opposite influences of the gradient tending to force the particles to the westward and the earth's deflective force tending to swerve them eastward. It is obvious that allowing for an increase in gradient with increasing latitude, the condition of steady motion is realized and the air particles move over a wide range of latitude with uniform velocity. The principle of the preservation of areas is not thereby invalidated nor is it necessary to assume that retarding or damping influences operate to reduce the increased velocities called for by the change of latitude. The increase of relative easterly velocity due to increase of latitude is exactly neutralized by the increase of westerly velocity due to the gradient.

NOTES, ABSTRACTS, AND REVIEWS.

METEOROLOGICAL INFLUENCES OF THE SUN AND THE ATLANTIC.[1]

By Prof. J. W. Gregory, F. R. S.

[Reprinted from *Nature* (London), Aug. 5, 1920, pp. 715–716.]

The prospects of long-period weather forecasting and the explanation of major variations of climate appear to rest on two lines of investigation. The effort of the first is to connect changes in the weather with those in oceanic circulation; the second attributes the changes to variations in the heat supply of the sun acting through the atmospheric circulation. Each theory has its own *a priori* probability. The oceanic control of climate has the attraction that each ocean is a potential refrigerator, since it is a reservoir of almost ice-cold water, which, if

raised to the surface, must chill the air, disturb the winds, and enable polar ice to drift further into the temperate seas. Hence Meinardus, for example, connected the range of ice in the Icelandic seas and harvests in Germany with variations in the surface waters of the North Atlantic. The alternative theory has the recommendation that, since the earth receives its heat supply from the sun, variation in solar activity is the natural cause of climatic change.

The oceanic theory must be true in part. The abnormal character of some coastal climates is clearly due to the upwelling of cold water under the influence of offshore winds. Moreover, unusual spells of weather on some of the coasts and islands of the Atlantic follow changes in the quality of its surface water, as proved by Dr. H. N. Dickson for Northwestern Europe, and by Prof H. H. Hildebrandsson's demonstration that for 15 years there has been constant coincidence between rainfall in British Columbia and the weather in the following autumn in the Azores. The alternative theory that the

[1] Björn Helland-Hansen and Fridtjof Nansen, "Temperature Variations in the North Atlantic Ocean and in the Atmosphere." Introductory Studies on the Cause of Climatological Variations. Smithsonian Miscellaneous Collections, vol. lxx., Publication 2537. 1920. Pp. viii+408+48 plates.
Author's abstract of original work in German, published in MONTHLY WEATHER REVIEW, April, 1918, 46: 177–178.